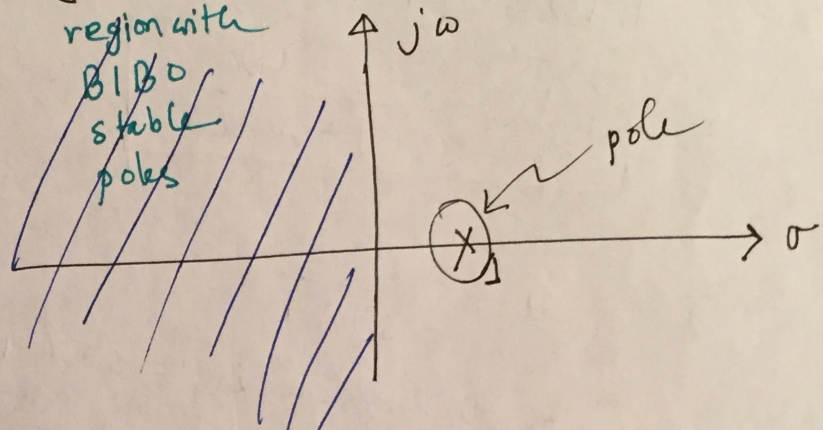
7.30/ $h(t) = e^{t}\, u(t)$ [Ref: page # 375-377 Philips, Parr 4th ed]

$\Rightarrow H(s) = \frac{1}{s-1}$; ROC: $\mathrm{Re}(s) > 1$

(a) For the given system $h(t) = e^{t}\, u(t)$
the pole $(s-1)$ resides on the right hand side
of the s-plane (w.r.t. $\sigma = 0$)

$\Rightarrow$ the system is not
BIBO stable.



e.g. let's consider $x(t) = u(t)$ [bounded input]

$$\Rightarrow y(t) = e^{t} u(t) * u(t)$$
$$= \frac{1-e^{t}}{-1} u(t) = (e^{t}-1)\, u(t)$$

when $t \to \infty \Rightarrow y(t) \to e^{t} \to \infty$. [undounded output]

(b)

From Fig 2.

$W(s) = X(s) - A\,Y(s)$ — (I)

and $Y(s) = W(s)\, H(s)$ — (II)

now, the system transfer function
with feedback is

$\hat{H}(s) = \frac{Y(s)}{X(s)}$ — (III) (Fig 3)

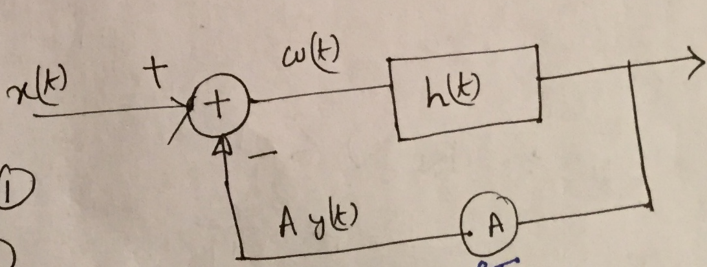


Fig 1.

$\Downarrow$ $\mathcal{LT}$

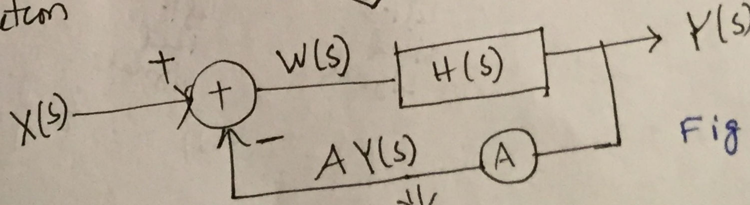


Fig 2.

$\Downarrow$

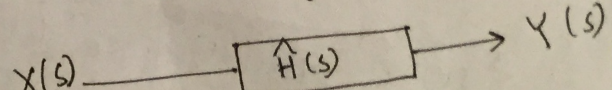


Fig 3.

From (ii) $\Rightarrow$

$$Y(s) = W(s)\,H(s)$$

$$\Rightarrow Y(s) = [X(s) - A\,Y(s)]\;H(s)$$

$$\Rightarrow Y(s)/X(s) = [1 - A\;Y(s)/X(s)]\;H(s)$$

$$\Rightarrow \hat{H}(s) = [1 - A\,\hat{H}(s)]\;H(s)$$

$$\Rightarrow \boxed{\hat{H}(s) = \frac{H(s)}{1 + A\,H(s)}} \Rightarrow \hat{H}(s) = \frac{1/s-1}{1 + A/s-1} = \frac{1}{s+A-1}$$

(c) $\hat{H}(s) = \dfrac{1}{s+A-1}$

pole of the system is : $s + A - 1$

for ~~...~~ BIBO stable system $Re(1-A) < 0$

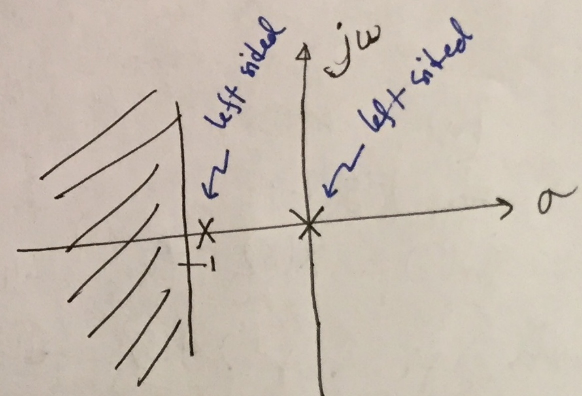
$$\Rightarrow 1 - R(A) < 0$$

$$\Rightarrow R(A) > 1.$$

~~7.24~~

$$F(s) = \frac{s+9}{s(s+1)} = \frac{0+9}{s(1)} + \frac{-1+9}{(s+1)(-1)}$$

$$= \frac{9}{s} - \frac{8}{s+1}$$

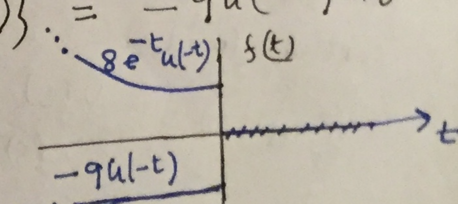


(a) ROC : $Re(s) < -1$

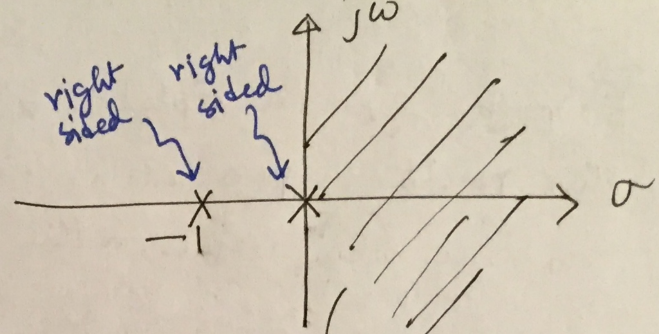
$$F(s) = \underbrace{\frac{9}{s}}_{\text{left sided signal}} - \underbrace{\frac{8}{s+1}}_{\text{left sided signal}}$$

$$\Rightarrow f(t) = 9\{-u(-t)\} - 8\{-e^{-t}u(-t)\} = -9u(-t) + 8e^{-t}u(-t)$$

$$\Rightarrow \boxed{f(t) = -9u(-t) + 8e^{-t}u(-t)}$$

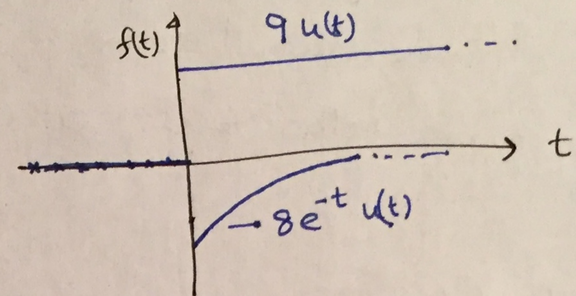


(b) ROC: $\mathrm{Re}(s) > 0$

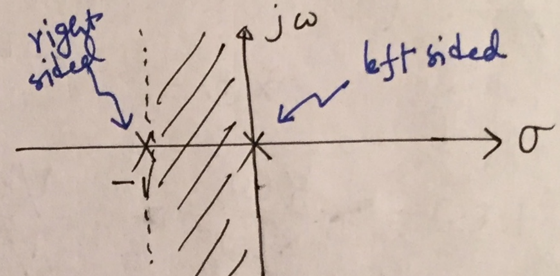


$$F(s) = \underbrace{\frac{9}{s}}_{\text{right sided signal}} - \underbrace{\frac{8}{s+1}}_{\text{right sided signal}}$$

$$\Rightarrow \boxed{f(t) = 9\,u(t) - 8e^{-t}\,u(t)}$$



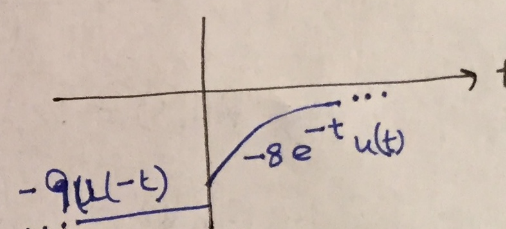
(c) ROC: $-1 < \mathrm{Re}(s) < 0$



$$F(s) = \underbrace{\frac{9}{s}}_{\text{left sided signal}} - \underbrace{\frac{8}{s+1}}_{\text{right sided signal}}$$

$$\Rightarrow f(t) = 9\{u(-t)\} - 8e^{-t}u(t)$$

$$= -9u(-t) - 8e^{-t}u(t)$$



(d) From final value theorem:

$$x(\infty) = \lim_{s\to 0} sX(s) = \lim_{s\to 0} \frac{s\cdot(s+9)}{s(s+1)}$$

$$= \lim_{s\to 0} \frac{s+9}{s+1} = 9$$

For (a) from the plot $x(\infty) = 0$ for ROC: $\mathrm{Re}(s) < -1$

Final value theorem is not applicable as the poles reside on R.H.S of the R.O.C.

For (b) from the plot $x(\infty) = 9$.
This result agrees with final value theorem as the poles reside on the left hand side of the R.O.C.

For (c) from the plot $x(\infty) \Rightarrow 0$.
Final value theorem is not applicable for this R.O.C as poles reside either side of the R.O.C.

7.20/ (a) $y(t) = e^{-bt} u(t) \Rightarrow Y(s) = \frac{1}{s+b}$

$x(t) = e^{-at} u(t) \Rightarrow X(s) = \frac{1}{s+a}$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1/s+b}{1/s+a} = \frac{s+a}{s+b}$$

$$= \frac{(s+b) - b + a}{(s+b)}$$

$$\Rightarrow H(s) = 1 + \frac{a-b}{s+b}$$

$$\Rightarrow \boxed{h(t) = \delta(t) + (a-b)\, e^{-bt}\, u(t)}$$

verification: $y(t) = h(t) * x(t)$

$$= [\delta(t) + (a-b) e^{-bt} u(t)] * e^{-at} u(t)$$

$$= e^{-at} u(t) + (a-b) \frac{e^{-bt} - e^{-at}}{(a-b)} \cdot u(t)$$

[from convolution table attached]

$$\Rightarrow \boxed{y(t) = e^{-bt} u(t).}$$

(b) $x(t) = u(t) \Rightarrow X(s) = \frac{1}{s}.$

$$y(t) = e^{-at}\cos(bt)\,u(t) \Rightarrow Y(s) = \frac{s+a}{(s+a)^2+b^2}$$

$$\Rightarrow H(s) = \frac{Y(s)}{X(s)} = \frac{s+a}{(s+a)^2+b^2} \cdot \frac{1}{1/s}$$

$$\Rightarrow H(s) = \frac{s(s+a)}{(s+a)^2+b^2} = \frac{s^2+as}{s^2+2as+a^2+b^2}$$

$$\Rightarrow H(s) = \frac{(s+a)^2+b^2-as-a^2-b^2}{(s+a)^2+b^2}$$

$$\Rightarrow H(s) = 1 - \frac{as+a^2+b^2}{(s+a)^2+b^2}$$

$$\Rightarrow H(s) = 1 - a\frac{(s+a)}{(s+a)^2+b^2} - b\cdot\frac{b}{(s+a)^2+b^2}$$

$$\Rightarrow \boxed{h(t) = \delta(t) - a e^{-at}\cos(bt)\,u(t) - b e^{-at}\sin(bt)\,u(t)}$$

<u>verification:</u>

$$h(t) = \delta(t) - a e^{-at}\cos(bt)\,u(t) - b e^{-at}\sin(bt)\,u(t)$$

$$= \delta(t) - e^{-at}\left[a\cos(bt) - b\sin(bt)\right]u(t)$$

$$= \delta(t) - e^{-at}\left[\cos(bt)\cdot\cos\theta - \sin(bt)\cdot\sin\theta\right]u(t)$$

assume,

$$\frac{a}{\sqrt{a^2+b^2}} = \cos\theta \qquad \theta = \tan^{-1}(-b/a)$$

$$\frac{-b}{\sqrt{a^2+b^2}} = -\sin\theta$$

$$\Rightarrow \boxed{h(t) = \delta(t) - \sqrt{a^2+b^2}\,e^{-at}\cos(bt+\theta)\,u(t)}$$

$$y(t) = h(t) * x(t)$$

$$= \left[\delta(t) - \sqrt{a^2+b^2}\,e^{-at}\cos(bt+\theta)\,u(t)\right] * u(t)$$

$$= u(t) - \sqrt{a^2+b^2}\cdot\frac{\overbrace{\cos(\theta-\phi)}^{1}\,e^{\overbrace{\lambda t}^{1}} - e^{-at}\cos(bt+\theta-\phi)}{\sqrt{a^2+b^2}}\,u(t)$$

$$\left[\phi = \tan^{-1}(-b/a) = \theta \quad \lambda = 0\right]$$

$$\Rightarrow y(t) = e^{-at}\cos(bt)\,u(t)$$

TABLE 2.1 Convolution Table

No.	$x_1(t)$	$x_2(t)$	$x_1(t) * x_2(t) = x_2(t) * x_1(t)$
1	$x(t)$	$\delta(t-T)$	$x(t-T)$
2	$e^{\lambda t}u(t)$	$u(t)$	$\dfrac{1-e^{\lambda t}}{-\lambda}u(t)$
3	$u(t)$	$u(t)$	$tu(t)$
4	$e^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(t)$	$\dfrac{e^{\lambda_1 t}-e^{\lambda_2 t}}{\lambda_1-\lambda_2}u(t) \qquad \lambda_1 \neq \lambda_2$
5	$e^{\lambda t}u(t)$	$e^{\lambda t}u(t)$	$te^{\lambda t}u(t)$
6	$te^{\lambda t}u(t)$	$e^{\lambda t}u(t)$	$\frac{1}{2}t^2e^{\lambda t}u(t)$
7	$t^N u(t)$	$e^{\lambda t}u(t)$	$\dfrac{N!\,e^{\lambda t}}{\lambda^{N+1}}u(t) - \displaystyle\sum_{k=0}^{N}\dfrac{N!\,t^{N-k}}{\lambda^{k+1}(N-k)!}u(t)$
8	$t^M u(t)$	$t^N u(t)$	$\dfrac{M!N!}{(M+N+1)!}t^{M+N+1}u(t)$
9	$te^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(t)$	$\dfrac{e^{\lambda_2 t}-e^{\lambda_1 t}+(\lambda_1-\lambda_2)te^{\lambda_1 t}}{(\lambda_1-\lambda_2)^2}u(t)$
10	$t^M e^{\lambda t}u(t)$	$t^N e^{\lambda t}u(t)$	$\dfrac{M!\,N!}{(N+M+1)!}t^{M+N+1}e^{\lambda t}u(t)$
11	$t^M e^{\lambda_1 t}u(t)$	$t^N e^{\lambda_2 t}u(t)$	$\displaystyle\sum_{k=0}^{M}\dfrac{(-1)^k M!(N+k)!\,t^{M-k}e^{\lambda_1 t}}{k!(M-k)!(\lambda_1-\lambda_2)^{N+k+1}}u(t)$
	$\lambda_1 \neq \lambda_2$		$+\displaystyle\sum_{k=0}^{N}\dfrac{(-1)^k N!(M+k)!\,t^{N-k}e^{\lambda_2 t}}{k!(N-k)!(\lambda_2-\lambda_1)^{M+k+1}}u(t)$
12	$e^{-\alpha t}\cos(\beta t+\theta)u(t)$	$e^{\lambda t}u(t)$	$\dfrac{\cos(\theta-\phi)e^{\lambda t}-e^{-\alpha t}\cos(\beta t+\theta-\phi)}{\sqrt{(\alpha+\lambda)^2+\beta^2}}u(t)$
			$\phi = \tan^{-1}[-\beta/(\alpha+\lambda)]$
13	$e^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(-t)$	$\dfrac{e^{\lambda_1 t}u(t)+e^{\lambda_2 t}u(-t)}{\lambda_2-\lambda_1} \qquad \mathrm{Re}\,\lambda_2 > \mathrm{Re}\,\lambda_1$
14	$e^{\lambda_1 t}u(-t)$	$e^{\lambda_2 t}u(-t)$	$\dfrac{e^{\lambda_1 t}-e^{\lambda_2 t}}{\lambda_2-\lambda_1}u(-t)$